The Young Child and Mathematics

Second Edition

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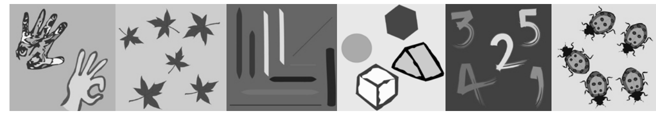

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Chapter 1

The Child Learns, the Child Teaches

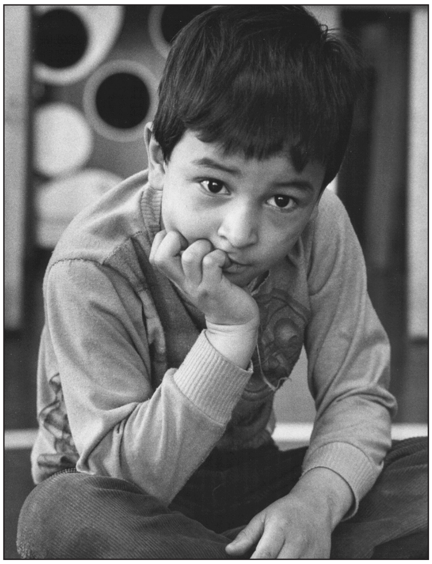
An 8-year-old with spiky hair, an impish demeanor, and a seemingly permanent pout, Timmy entered my life late one October. He was referred to me because he demonstrated few skills in reading or mathematics, seemed unmotivated to attempt new tasks, and seldom—if ever—completed a readable assignment.

Jessica, age 8, was quiet and compliant. She never caused problems, never showed any particular gift for mathematics, and never volunteered in class. In fact, I had difficulty finding anything specific to say to her parents during parent-teacher conferences.

With hands up in the air and a grin a mile wide, 5-year-old Armand, an enthusiastic ball of energy, entered the science lab with his kindergarten class and announced, "I'm here! I love science! I can't wait!" Within an hour I found Armand on the floor holding an earthworm, surrounded by dirt from the earthworm farm, every inch of bare skin and most of his clothing covered with dirt.

I have been a teacher for 38 years. I have taken courses, read books, earned degrees, conducted research, and published articles. I have learned how to do mathematics, how to teach mathematics, how to assess mathematical understanding, how to understand child development, and more. My teachers and mentors have been wonderful. They have taught me much, given me ideas, and broadened my beliefs about teaching. However, the lessons I learned from children—from Timmy, Jessica, Armand, and others—are the ones that have made the most difference in my teaching.

The Young Child and Mathematics focuses on children from age 3 through 8 and their mathematical learning. The placement of the phrase *young child* before the word *mathematics* in the book's title is not accidental. It comes first because I believe that the child should be the focus of early education. In this chapter I share some of the lessons that I have learned about children and their understanding of mathematics. Some of my ideas about how to teach mathematics come from textbooks, some from watching other teachers teach, and some from experts. However, most, if not all, of the ideas were greatly influenced by children themselves. In some cases their strategies changed the activities for my lessons, their interests changed the scope of the content, or their particular strengths or weaknesses changed the sequence of my instruction. Whatever the case, I have found that when I make the child the focus of my teaching, I teach mathematics well.



Learning from Timmy

Let's talk about Timmy. After looking at his work and his records, I understood his teacher's frustration. Indeed, Timmy was a child with whom teachers found it difficult to work. He had demonstrated a very short attention span, and he showed little promise in mathematics or any other subject. I admitted him to a special program for "slow learners."

Because of an unexpected assignment, I didn't have time to work with Timmy that first day he appeared in my classroom. To keep him busy, I gave him a large box of electrical equipment from the sixth grade electricity unit: switches, batteries, wire, and small light bulbs. I asked him to sort the box's contents and told him that I would return in a while.

Thirty minutes later, when I walked back into the room, I discovered that Timmy had created a working electrical system! Six bulbs were lit, three switches were incorporated, and within the connection he had produced a working model of both a parallel and a series circuit. I watched and listened as this challenging, unmotivated, and unskilled 8-year-old shared with me his creation, which was much more complicated than anything I could build. Timmy explained why certain bulbs were half the brightness of other bulbs, he told me how to predict whether a bulb would light or not light, and he told me about resistance and the advantages and disadvantages of series and parallel circuits.

Timmy taught me an important lesson: Spend time observing, listening, and watching children. Pay attention to what they like, listen to their reasoning, ask them to explain their creations, challenge them with tasks that seem impossible, and give them the opportunity to show you what they can do in the way they want. If I encourage children in these ways, I provide them a greater opportunity to reach their true potential in mathematics!

A lesson from Jessica

Quiet Jessica was one of my students during my second year of teaching. At the end of the year, I asked each child in my class to give me a report card for being a teacher. I stressed the fact that they should tell me good things as well as things I could improve.

Jessica took my directions seriously. Her block-print note with her own spelling stated,

> Ms Copley. You were real good with the dumd kids. They needed you and you halped them. You were real good with smart kids. You always keeped them buzy. but you should do better with the plane kids like me. I need to learn to!
>
> Luv, Jessica

She was right; I had spent most of my time with the special children, and I had ignored the "plain" kids. I learned an important lesson from that note: Remember that every child is important! It is my job to do my best teaching for those children who have special gifts; those who need concentrated help to overcome difficulties; *and* those quiet, plain children who have the right to learn.

The joy of Armand

Armand's fascination with earthworms began that first day in the science lab. He spent countless hours observing them, "reading" earthworm books in the library, and asking questions. During his study he became obsessed with finding the eyes of earthworms. Assuring me that "They gots to have eyes! They gots to see!" Armand kept asking for bigger magnifiers so that he could find their eyes.

Instead of discouraging Armand and correcting his misconception, I helped him set up experiments to test his hypothesis. During the last two months of the school year, Armand spent every free minute conducting experiments with colored mats, homemade earthworm houses, and colored lights. As he boarded the bus for his final trip home as a kindergartner, he yelled out the window, "I still think they gots eyes, Mrs. Copley!"

That statement taught me a great deal. Because I allowed him to investigate his hypothesis—and in fact encouraged his exploration—Armand learned more about earthworms than anyone in the school. Because he had a need to know, Armand read fourth and fifth grade–level books about earthworms and could use the proper terminology to describe their body parts. More important, because he was in charge of his own learning, Armand continued to be a powerful, excited learner.

When I heard Armand's final statement, I was reminded how important it is not to jump to correct every misconception. Instead, encourage investigation, and remember that children construct their own knowledge. What an valuable lesson for someone teaching mathematics!

My three lessons

These three lessons should be in evidence throughout this book. First, spend time observing, listening, and watching children. Some of the many vignettes and dialogues I have recorded, videotaped, and remembered appear in every chapter. I believe that you can learn most from children and classroom examples; thus, such examples are abundant and presented as realistically as possible.

Second, remember that every child is important. This volume offers a variety of examples and suggested activities that work for children with all types of needs in diverse settings. Because I have been privileged to teach in multicultural settings; in urban and rural communities; in private and public schools; and in 4-year-old, kindergarten, first grade, and second grade classrooms, the ideas here reflect those contexts.

Third, encourage investigation and remember that children construct their own knowledge. I value the joy of learning, exploration, and discovery. While mathematics is often considered a subject of right answers and prescriptive instruction, the ideas presented in this book foster investigation in the way children learn.

1. The intuitive mathematical knowledge of the young child

Young children are natural learners. They construct their own understanding about quantity, relationships, and symbols. They approach new tasks with curiosity and a sense of experimentation. Counting is a natural task, *more* is a word 2-year-olds know readily, and the process of adding and subtracting can describe and explain to children situations that they encounter in their world. When a new idea or piece of information doesn't make sense to a child, Jean Piaget theorized that the experience creates dissonance—mental conflict that the child seeks to resolve. Thus, the child develops and assimilates knowledge, making it her own.

The intuitive, informal mathematical knowledge of young children often surprises early childhood teachers. Yet, kindergarten curriculum tends to reflect the belief that 5-year-olds enter school as blank slates, with no mathematics concepts and no experience with quantities, patterns, shapes, or relationships! Instead, research strongly indicates that young children have a strong, intuitive understanding of informal mathematics.

Block Shapes

Three-year-old Jeffrey discusses a new set of blocks with his teacher:

Ms. Wright: Tell me about these new blocks. What do you call them?

Jeffrey: Blocks with different shapes.

Ms. Wright: What can you tell me about these different shapes?

Jeffrey: This yellow one is a star. The blue one is a triangle.

Ms. Wright: Wow! Have you ever seen shapes like this before?

Jeffrey: [sighing loudly] Yeah, at my house, my Granna house, my Daddy house, outside.

Ms. Wright: You have all these shapes at everybody's house?

Jeffrey: No. . . . [picking up the orange circle] This is like my Uncle Dee's basketball, but [frowning] it won't bounce up and down. This is a piece of pizza [indicating a purple, wedge-shape block], this is a table [a pink square], this is my best book [a rectangular block].

Jeffrey proceeds to separate the blocks into two groups: five shapes on one side and one shape on the other side. The star-shaped block is alone, and the others—triangle, square, pizza slice, circle, and rectangle—are heaped together in a pile.

Ms. Wright: Why did you put the star on the other side?

Jeffrey: [with a deep, long sigh that sounds as if he has lost his patience] These belong in a house and this one [the star] doesn't. It belongs in the sky!

Did you notice Jeffrey's verbal labeling of the block shapes, his connection of the blocks to items he regularly sees, and his clarification of differences between the basketball and the round block? While his particular classification system (belongs in the house, doesn't belong in the house) is not found in state objectives, his system is perfectly correct and indicates greater consistency than young children typically show.

Identification of two-dimensional shapes is an objective in kindergarten programs; Jeffrey has already shown that ability at age 3. The identification of shapes in the everyday world is also a standard objective in kindergarten curricula; again Jeffrey demonstrates that ability. I am not proposing that all 3-year-olds have Jeffrey's understanding and use of language; however, I do believe that many early childhood programs and teachers view young children as incapable, when in fact they already grasp many mathematical concepts at an intuitive level.

While reviewing research studies is not the purpose of this book, I have listed below some important points supported by research (Copley 1999; Clements & Sarama 2007; NRC 2009) that illustrate the intuitive mathematical knowledge of the young child:

- Young children have the ability to learn mathematics. Through everyday experiences they acquire a wealth of informal knowledge and strategies to deal with situations that have a mathematical dimension.

- The operations of addition, subtraction, multiplication, and division are often understood by young children. While they may not be able to complete a written equation such as $5 - 2 = 3$, they can easily tell you how many buttons you would have on your shirt if you started with 5 and 2 fell off. Accordingly, they can figure out how many pieces of candy to purchase for a birthday party if everyone attending got 3 pieces.

- Children's understanding of rational numbers, while incomplete, is often more accomplished than expected. Their common sharing experiences, their use of the term *half*, and their fair distribution of quantities among friends are natural by-products of everyday experiences.

- The development of geometric concepts and spatial sense can often be observed when young children participate in free play. The young child directing a building block project uses words and motions to tell his friend how to make a castle. “Do it like me. You need a square block” (his description for a cube). “No, not that way. Turn it over!” When his friend says, “It doesn’t matter, it’s always the same,” the child reaffirms his own understanding by saying, “Well that’s ’cause now it’s right!” Both children are experimenting with beginning concepts of rotation and the language of geometry.
- A natural fascination with large numbers is evident with young children. While they frequently invent nonsensical numbers (a million-dillion-killion), they often show a partial understanding of quantity and the need for counting.

Do we need to directly teach young children all they need to know about mathematics? Do we need to start from the beginning, drill in those basic facts, and fill all the holes in their understanding? Do we need to tightly define as developmentally appropriate only very easy mathematical concepts?

The answer to all of these questions is a resounding *No!* Instead, we need to remember that young children possess a vast amount of intuitive, informal mathematical knowledge. Our job is to assess their prior knowledge, build upon their strengths, facilitate their learning, and enjoy the process.

2. The constructed mathematical concepts of the young child

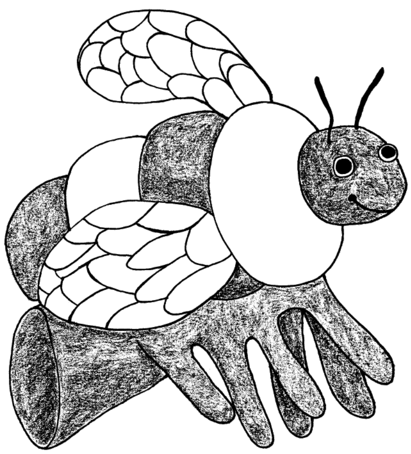
Young children continually construct mathematical ideas based on their experiences with their environment, their interactions with adults and other children, and their daily observations. These ideas are unique to each child and vary greatly among children the same age. Some of the ideas are perceptually immature, many of the problem-solving strategies are inefficient, and the verbal information necessary to discuss mathematics is often incorrectly labeled or modeled. The child who perceives more candy to be in the larger bag, the child who always adds together two sets of items using the counting-all strategy, and the child who counts “6, 7, 8, 9, 10, oneteen, twoteen, threeteen,” all illustrate mathematical ideas that would be labeled incorrect for an adult but are developmentally correct for a young child.

An early childhood teacher who frequently listens to ideas expressed by young children can provide materials and an environment conducive to the development of mathematical concepts. More important, by observing young children, the teacher can ask questions that prompt them to make new discoveries and form their own questions. To illustrate this point, the vignette below shows how some prekindergartners respond to a problem presented by their classroom teacher.

Six Legs, Five Fingers

Miss Riley: I just bought this bee puppet [shows a bee puppet attached to a black, six-fingered glove] and I have a problem. When I put my hand into the glove, I always have an empty leg on the puppet. I don’t have enough fingers for all of the legs. I have only five fingers and there are six legs on this puppet. I don’t know what to do! Maybe I’ll just take it back to the store.

The children seem to be thinking about this problem. Some count their fingers, others talk to their partners, and still others shrug their shoulders.



Russell: Well, maybe you got your fingers in wrong.

Miss Riley: Maybe. Let's see. [The teacher puts her hand in and out of the puppet a few times, each time showing the leg without the finger.] No, it's still there.

Marta: Just cut it off.

Miss Riley: Well, that's an idea, but it's brand new. I don't want to cut up a brand new puppet!

Svetlana: I know. Let me show you. [Svetlana tries to put one of the legs inside the glove. It leaves a hole, and Svetlana shrugs and sits down.]

Miss Riley: Good try, Svetlana, but it still seems like it's not quite right.

Duane: Hey, maybe the guy who made it had six fingers!

This idea seems to satisfy many of the children, who nod. The children are excited about a possible solution.

Miss Riley: Maybe so. Let's see. How many fingers do you have on one hand?

Everyone spends time counting and recounting the fingers on their hands. Some repeat the counting three or four times.

Miss Riley: Does anyone have six fingers on a hand?

The children shake their heads no or respond verbally, saying they have five fingers. Others act like they do have six fingers, count aloud as they touch them, and then say that they have only five. After a few minutes they seem to be satisfied that no one in the class has six fingers.

Linda: [excitedly raising her hand] Umm . . . umm . . . maybe bees have six legs!

Duton: No, I have lots of bees at my house and they all have five legs! [Linda looks disappointed; because Duton is a class leader, most students believe he is right.]

Miss Riley: Maybe Linda has a good idea. How could we find out?

The children suggest many ideas, including going to the library center. After a brief discussion, the children disperse to different activity centers, some looking for books with bee pictures and others becoming involved in other activities. Angelica is still sitting, counting her fingers over and over again.

Angelica: [calling out after about five minutes] Teacher, teacher, look! If I count my fingers real fast, I get six!

This brief classroom vignette illustrates beautifully the variability and wonderful creativeness of young children as they attempt to solve a real problem. Russell's notion—that if the teacher's fingers are taken out and put back into the glove again, there would no longer be an extra leg—demonstrates a lack of number conservation, a characteristic common in many 4-year-olds. Russell seems to believe that if the appearance of the fingers in the puppet changes, the number of legs will change as well. Marta's and Svetlana's subtraction methods were expedient yet did not take into account any reason for the extra leg. Duane's creative idea about six-fingered people may indicate his lack of experience with other people's number of fingers. The class's ready belief that Duton's five-legged house bees are like all bees reveals faulty reasoning based on insufficient evidence. Finally, Angelica's humorous response illustrates an inefficient and incorrect "fast-counting" strategy and also reveals her ability to persist in seeking solutions.

Many research studies report findings consistent with the idea that children construct their mathematical knowledge through experiences. In addition, studies have suggested that the teacher's significant roles of observing, facilitating, supporting, and questioning are essential to that construction.

- Mathematics is for *everyone*. If mathematics is taught properly at the early childhood and elementary levels, all children should develop proficiency in it. Instances that appear to be learning disabilities in mathematics are often caused by inappropriate teaching rather than intellectual inadequacy.
- Few, if any, differences in young children's ability to learn mathematics relate to gender or socioeconomic status. Rather, *opportunity* to learn is the primary factor in the development of that ability.
- Young children make sense of mathematical situations in different ways. Not all children in a group represent or solve problems in the same way. Not all children follow the same specific developmental sequence. However, there are some *general trajectories or paths* that children travel and that teachers can follow that will give children experiences with a variety of thinking strategies and modeling procedures. (See Appendix B for a chart of these learning trajectories.)

Should we immediately correct young children's misconceptions about mathematics? Can we expect all children to solve problems in identical ways? Should we expect all the young children in a group to "get it" at the same time? Again, the answer to all these questions is *No!* As teachers, we need to remember that young children construct mathematical understanding in different ways, at different times, and with different materials. Our job is to provide an environment in which all children can learn mathematics.

3. The power of positive attitude

In 1990 President George H. W. Bush and the nation's governors established a list of national education goals to be reached in the year 2000. Goal one states, "By the year 2000, all children in America will start school ready to learn." While I understand that the panel was concerned about the physical, social, and emotional needs of the young child, my experience, supported by countless research studies, has demonstrated that the young child has long been more than ready to learn mathematics. In fact, by the time she starts school, she is already learning more complex mathematics than we might expect.

Young children are typically motivated to learn quantitative and spatial information. Their dispositions allow them to be positive and confident in their mathematical abilities. The highly prized characteristics of persistence, focused participation, hypothesis testing, risk taking, and self-regulation are often present, but seldom acknowledged, in the young child.

After describing a young child's strong motivation to learn mathematics, the National Research Council (1989; 2001) stated that a gradual change occurs in the early primary grades and described it as a shifting disposition from enthusiasm to apprehension and from confidence to fear. Unfortunately, this gradual change is often completed by third grade. In *Adding It Up*, members of the Council stated,

> Most U.S. children enter school eager to learn and with positive attitudes toward mathematics. It is critical that they encounter good mathematics teaching in the early grades.

> Otherwise, those positive attitudes may turn sour as they come to see themselves as poor learners and mathematics as nonsensical, arbitrary, and impossible to learn except by rote memorization. (NRC 2001, 132)

After spending two years researching the motivation of 7- and 8-year-olds as they solve spatial tasks, I was amazed at the differences in motivation of children in that age group. Let me illustrate the two contrasting situations by describing two children's actions during a spatial task.

According to the cognitive ability measure in the example below, Roberto and Mei-Chi possessed almost the same analytical and quantitative ability. English was the first language for both children, and they both participated in a program for gifted students. To assess their specific motivation to solve problems, each child was given the choice of doing "easy" or "hard" symmetry tasks during a ten-minute session. Each task consisted of asking the child to place a mirror (held vertically) adjacent to a picture card (laid flat on the table) such that the image created by the picture and its reflection in the mirror looked like the image on the task card presented by the teacher. Unknown to the children, the easy tasks were quite easy and could be solved in less than fifteen seconds by the average 7-year-old, and the hard tasks were impossible because of the target image's nonsymmetrical characteristics.

See how Roberto's and Mei-Chi's responses differ in this problem-solving task:

A Matter of Motivation

Mrs. Evans: Mei-Chi, you may choose to do either the hard tasks or the easy ones. I don't care which ones you do, and you may stop at any time and change to another task. I will tell you if you have solved the problem or if you haven't solved the problem. But you do not need to solve the problem to move on to another task. It will be your choice. Understand?

Mei-Chi: I will take an easy one.

The task card is given to Mei-Chi, and she solves it in approximately fifteen seconds. The teacher states that it is correct.

Mei-Chi: I will take another easy one.

Again, Mei-Chi quickly solves it, and the teacher states that it is correct.

The activity continues for ten minutes. Mei-Chi solves more than thirty-five easy tasks, never asking for a hard one. Each time she solves one, she looks at the teacher and asks if it is correct.

Mrs. Evans: Mei-Chi, before you go . . . I noticed that you never tried a hard task. I was wondering why not. Could you tell me?

Mei-Chi: Yeah, I might get it wrong! You told me they are hard!

Mei-Chi leaves and Roberto enters. The same directions are given to Roberto.

Roberto: I will take a hard one.

Roberto eagerly takes the task card. He maneuvers the card and mirror, trying to complete the task in a variety of ways. Roberto works on the one hard task during the entire ten minutes, ignoring the teacher and never completing the task.

Mrs. Evans: Roberto, I am sorry, our time is up. You will need to leave the task here for now and go back to class.

Roberto: Aw, I was just about to get it. Can't I stay a little longer?

Mrs. Evans: No, I'm sorry. I promise to let you try again on another day.

As Roberto starts to leave, the teacher is busy preparing for another student. She glances up and sees Roberto placing the task card in his back pocket.

Mrs. Evans: Roberto, I need that card. I'm sorry, you will need to leave it with me.

Roberto: If you would just let me take it home, I know I could get it!

What different responses! Mei-Chi completes many tasks successfully; Roberto never successfully completes one task. Mei-Chi requires a teacher's acknowledgment of success; Roberto seems to ignore the teacher. Mei-Chi attempts short, easy tasks; Roberto demonstrates great persistence on only one task. Mei-Chi takes no risks; Roberto even risks taking the task card against the teacher's wishes.

Both of these children are smart, capable mathematics learners. However, on this task they demonstrate quite different motivational characteristics. Mei-Chi worked for the teacher's approval, performing many tasks but never trying hard ones. Her motivation could be termed *performance-oriented*. Roberto, on the other hand, worked to complete the task, persisting at a hard task in spite of his continual failure. His motivation could be termed *task-learning–oriented*.

Research indicates that disposition is very important to the long-term learning of mathematics (Renga & Dalla 1993). Disposition concerns more than attitudes toward mathematics alone; persistence, risk taking, hypothesis making, and self-regulation are all important to a motivated disposition.

- All children seem strongly motivated to perform well in school mathematics. By about third grade, however, important differences in both motivation and achievement begin to emerge.
- Performance-oriented children are motivated by others' approval and often "perform" to be successful. They frequently underestimate their successes and overestimate their failures. In addition, they demonstrate little persistence and, when confronted with a problem, can exhibit a state of "learned helplessness."
- Task-learning–oriented children are motivated by learning and view each task as something to be mastered. They seldom express interest in what others think; in fact, they often ignore feedback about success or failure. They frequently take risks, demonstrate great persistence, and often continue a task in spite of many obstacles.
- Teachers can directly influence performance-oriented children. When teachers stress learning rather than performance, significant differences are observable in children's motivation toward mathematics.

Should we ignore children's dispositions toward mathematics? Should we continually stress the importance of teacher-pleasing behavior when doing mathematics? Should we assume that young children fear mathematics and are as anxious as many adults when learning mathematics? Clearly, we should *not!*

Young children are motivated to do mathematics. Our job as teachers is to stress the importance of learning, model the joy of mastering tasks, and value errors as essential information to help us learn.

About this edition

In 2000, the National Council of Teachers of Mathematics published its landmark guide *Principles and Standards for School Mathematics*, setting mathematics education, prekindergarten through twelfth grade, on a path toward an exciting but "highly ambitious" new vision of "mathematics classrooms where students of varied backgrounds and abilities work with expert teachers, learning important mathematical ideas with understanding, in environments that are equitable, challenging, supportive, and technologically equipped for the 21st century" (NCTM 2000, 4).

I wrote the first edition of *The Young Child and Mathematics* that same year, aiming for a practical book for teachers that communicated the main ideas of NCTM's Standards within the context of effective, developmentally appropriate practice. My book followed the structure of the Standards, too—one chapter on what young children should know and be able to do with the five "processes" of mathematics (Problem Solving; Reasoning and Proof; Communication; Connections; and Representation) and a chapter each on the five mathematics "content areas" (Number and Operations; Algebra; Geometry; Measurement; and Data Analysis and Probability).

This second edition of *The Young Child and Mathematics* is very similar in purpose and structure to the first. What's different is that in the intervening ten years, there have been a number of important developments in the field of early childhood math education. In particular, the following new resources have influenced my thinking and teaching, and their insights are reflected in this second edition:

- researched, field-tested curricula specifically written for preschool children, funded by the National Science Foundation;
- two research volumes by the National Research Council—*Adding It Up: Helping Children Learn* (2001) and *Mathematics Learning in Early Childhood: Paths toward Excellence and Equity* (2009);
- the NAEYC/NCTM joint position statement, Early Childhood Mathematics: Promoting Good Beginnings, adopted in 2002;
- NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006), which describes general mathematics expectations by grade level and outlines which objectives teachers should give the most attention; and
- *Learning and Teaching Early Math: The Learning Trajectories Approach* (Clements & Sarama 2009), which details children's learning paths, including developmental information by content area.

But for me, the most important developments over the last decade were the many new experiences I have had with young children in very diverse settings: Head Start centers, public prekindergarten programs, private and public school settings, children's museums, child care centers—and most exciting for me, discoveries with my four young grandsons!

The chapters at a glance

This first chapter has introduced you to young children and their characteristics as learners of mathematics—and as sources of learning for teachers, too! Let me restate the understandings expressed in this chapter, including their implications for teaching, in their most simplified form:

- Young children
  - possess a large amount of intuitive mathematical knowledge;

– continually construct mathematical ideas based on their experiences and observations; and

– are strongly motivated to do mathematics.

- Teachers of young children should

– encourage investigation of and experimentation with mathematical concepts;

– spend time observing, listening, and watching children; and

– remember that *every* child is important.

Throughout the book these ideas will be illustrated through my descriptions of classroom situations, children's work products, and their interactions with one another and with adults.

Chapter 2 describes the framework that has guided me in my own work teaching and learning from young children. It consists of twelve guidelines for *curriculum, instruction,* and *assessment*—the three components of teaching. Further, the guidelines integrate what the National Association for the Education of Young Children suggested in its most recent statement on developmentally appropriate practice (NAEYC 2009) with NCTM's Focal Points (2006).

Like the first edition, the major portion of this book is six chapters that describe what young children should know about and be able to do with mathematics. Chapter 3 focuses on the five essential *processes* of mathematics. Chapters 4 through 8 focus on the five essential *content areas* of mathematics. In line with my belief about what makes for effective professional development (see the Question box opposite), the chapters on mathematics processes and content were written to connect teachers' knowledge and understanding with national curriculum standards and with children's learning paths.

Across my many, many conversations with teachers in classrooms and in trainings over the years, I often have been asked the same questions about teaching mathematics to young children. Some of these questions are answered in boxes sprinkled throughout the book. Other frequently asked questions, particularly those whose answers send readers elsewhere, I have collected in chapter 9.

This book contains two appendices. Appendix A provides instructions for accessing the accompanying DVD. Appendix B is a chart of learning trajectories for each of the content areas discussed in this book, originally from the NAEYC/NCTM joint position statement.

Finally, this 2010 book is accompanied by a DVD, on which NAEYC and I offer a menu of useful resources, including an expanded children's book list, the 2002 NAEYC/NCTM math position statement, and NCTM's Standards and Focal Points for prekindergarten through second grade. Most important, the DVD provides video clips from my own work in classrooms, demonstrating mathematics education in action and also giving a visual accompaniment to several of the activities I suggest in the content chapters.

*   *   *

I am constantly intrigued by young children, their characteristics, and their ways of learning mathematics. I hope as you read this book that my excitement is communicated and ignites your interest and efforts in educating the young child.

Q

I'm a director of a preschool; what kind of mathematics professional development is best for my teachers?

Based on my experience teaching young children and my understanding of the research on adult learning, I believe the most effective professional development for teachers working with young children has seven critical elements:

The professional development fills any gaps in teachers' own *mathematics content knowledge* (Ma 1999; Ball & Bass 2000) and their *understanding of child development* or learning paths in specific mathematics areas. It helps teachers *make connections* between national standards (e.g., NCTM's Focal Points) and state/local learning outcome requirements. It describes *specific mathematics activities* for teachers to use in their classroom routines, small-group instruction, whole-group activities, and centers. It identifies which *instructional strategies* are particularly effective in mathematics. It *models activities* for teachers, as video clips and/or live in-class demonstrations. And it offers *coaching* that provides follow-up and specific goals.

This type of professional development is intensive and can be costly. But if mathematics is foundational to a young child's learning, and if we are to teach it with intentionality in early childhood classrooms, all these elements are necessary!