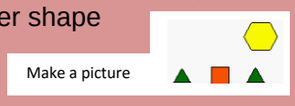
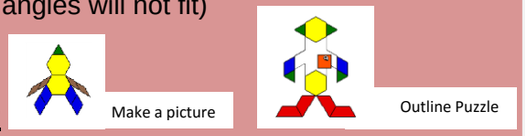
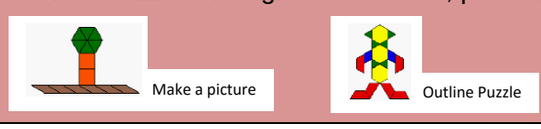
HANDOUT 3:
Math Toolbox Trajectories: Language and Games

Learning Trajectories

Age	Shapes	Composing
2	**Shape Matcher (Basic)** Match shapes with BOTH Same size and orientation AND Different sizes and orientations	
3	**Shape Matcher** Match a wider variety of shapes with Same size and orientation AND Different sizes and orientations AND Combinations of shapes **Shape Prototype Recognizer and Identifier -** Recognize and name shapes	**Pre-composer** Manipulates shapes as individuals, but unable to combine them to compose a larger shape **Pre-decomposer**  Decomposes only by trial and error (Given only a hexagon, can break it apart to make this simple picture, by trial and error)
4	**Shape Recognizer (Basic1)** Recognizes non-prototypical squares and triangles and some rectangles Does not recognize rhombi (diamonds) Does not differentiate sides and corners **Constructor of Shapes from Parts** Representing: Makes a shape "look like" a goal shape (make triangle with sticks)	**Piece assembler** Makes pictures in which each shape represents a unique role and shapes touch (one shape for each body part) Fills simple outline puzzles using trial and error.
5	**Shape Recognizer (Basic 2)** Recognizes : ♦ more rectangle sizes, ♦ more shapes, ♦ more orientations of rectangles Correctly names shapes as rectangles **Side recognizer** Identifies sides as distinct geometric objects **Angle Matcher** Matches angles correctly **Shape Recognizer (3)** Recognizes most basic shapes and prototypical examples of other shapes (hexagon, diamond, trapezoid)	**Picture Maker** Puts several shapes together to make one part of a picture (two shapes for one arm) Uses trial and error and does not anticipate creation of new geometric shape Chooses shapes using "general shape" or side length Fills "easy" outline puzzles that suggest placement of each shape (note below that child is trying to put a square in the puzzle where its right angles will not fit) **Simple Decomposer**  Decomposes simple shapes that have obvious clues to their decomposition -given hexagons, can break apart to make this picture: Composes shapes with anticipation ("I know it will fit!") Chooses shapes using angles as well as side lengths **Shape Decomposer** Rotation and flipping are used intentionally to select and place shapes. Outline Puzzle: all angles are correct, patterning is evident 

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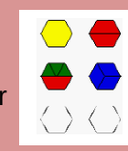
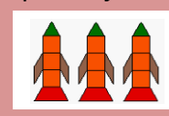
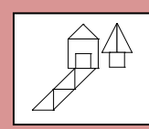
6	**Shape Identifier** Names most common shapes, including rhombi (“ellipse-is-not-circle”) **Angle Recognizer-**Implicitly recognizes right angles- distinguishes between rectangle and parallelogram without right angles. Recognizes angles as separate geometric objects.	**Substitution Composer** Makes new shapes out of smaller shapes  Uses trial and error to substitute groups of shapes for other shapes to create new shapes in different ways Outline Puzzle: substitutes shapes to fill different ways: **Shape Decomposer (with help)-**Decomposes shapes using imagery that is suggested and supported by the task or environment Given hexagons, can break it apart to make this shape:
7	**Parts of Shapes Identifier** Identifies shapes in terms of their components **Constructor of shapes from parts** Representing: Completely correct construction, based on knowledge of components and relationships	**Shape Composite Repeater** Constructs and duplicates units of units (shapes made from other shapes) intentionally Understands each as being both multiple small shapes and one large shape. May continue a pattern of shapes that leads to tiling  **Shape Decomposer with Imagery** Decomposes shapes flexibly using independently generated imagery**;** Given hexagons, can break apart to make shapes:
8	**Shape class identifier-**Uses class membership not explicitly based on properties (e.g. to sort- “I put triangles over here, quadrilaterals over there”) **Shape property comparer-**Uses properties explicitly **Angle size comparer-**Separates and compares angle sizes **Angle measurer-**Measures angles- uses protractor to measure angles of a shape **Property class identifier**- Uses class membership for shapes explicitly based on properties including angle measure (e.g. to sort or consider shapes similar-“I put equilateral triangles over here and right triangles over there”) **Angle synthesizer-**Combines various meanings of angle (turn, corner, slant)	**Shape Composer-Units of Units** Builds and applies units of units. (E.g. in constructing special patterns, extend patterning activity to create a tiling with new unit shape) Unit of unit shape that they recognize and continuously construct. **Shape Decomposer-Units of Units** Decompose shapes flexibly using independently generated imagery and planned decompositions of shapes that themselves are decompositions. Given only squares, can break them apart-and then break the resulting shapes apart again to make something like this: 

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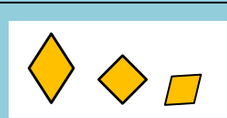
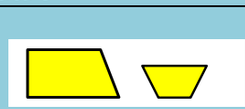
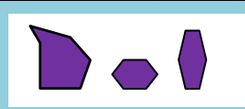
Shape Language: DESCRIBING AND TALKING ABOUT SHAPES

Shape Description	What to Say	What Not to Say
Triangle: • 3 straight sides and 3 angles • Different sizes, shapes and orientations • Regular or equilateral: all sides same length and all angles same • *Isosceles*: 2 sides are the same length • *Right triangle*: one angle is "right"(90°)	— Triangles have 3 sides — all 3 sides are straight, not curved — all 3 sides are connected — triangles can be different sizes, point in different directions, some sides may be the same length, but they can all be different lengths — can be long and skinny	— 2 points at bottom, one at top — point in the middle — flat bottoms — pointy — like a piece of pizza — like the open triangle used in music class — like a cone shaped clown hat
Square: • 4 straight sides, all same length, and all 4 right angles • 2 pairs of parallel sides • Different sizes and orientations		— turned/slanted — one is a diamond
Rectangle: • 4 straight sides and 4 right angles • Opposite sides are same length • If all sides are the same length, it's also a square • 2 pairs of parallel lines • Different sizes, shapes, and orientations	— rectangles have 4 sides — all sides are straight, not curved — all sides are connected — the opposite sides are the same length — rectangles have 4 right angles — Rectangles can be different sizes or point in different directions, they can be long and skinny or they can be a square	— two long sides and two short sides — rectangles are long
Rhombus:  • 4 straight sides, all same length • 4 angles with opposite angles equal • 2 pairs of parallel lines • If all angles are right angles, it's also a square		
Trapezoid:  • 4 straight sides and 4 angles • 1 pair of parallel lines • *Isosceles:* non-parallel sides are same length • *Right*: one or more right angles		
Hexagon:  • 6 straight sides and 6 angles • *Regular*: all sides and angles equal		

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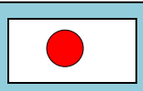
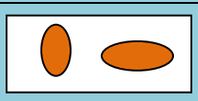
Circle  • A perfectly round shape • A line that is curved so that its ends meet and every point on the line is the same distance from the center		
Oval  • Having a rounded and slightly elongated outline or shape, like that of an egg		

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Shape Questions

1. How is that shape like this one? How is it different?
2. Why isn't this shape an oval (circle, square)? What makes it an oval?
3. What if I turned this shape or flipped it? What would it look like if I slid it from your paper to my paper?
4. Where have you seen this shape before?
5. Can you find something like this at home?
6. How did you decide what to copy/draw?
7. Can you tell me how to get to the cafeteria/ office/ playground from here?
8. Do you think this shape would roll? Slide? Could we stack these?
9. How could you cut this paper to make another shape?
10. What shape could you make out of these shapes?
11. Could we make the cone roll straight, or would it roll crooked? What about the cylinder?
12. Have you found all the ways to put those shapes together? How do you know?
13. What would happen if I cut off an end of this? What would it look like?
14. Can you think of another name for this shape?
15. Can you make a square/circle/triangle/ with pipe cleaners? How about a ball/box/cone?

Shape Games

1. **"2-D Faces of 3-D Shapes"-** *Using food boxes and cans of similar shapes but different sizes, paper and pencil. Trace different sides of the boxes and cans, shuffle papers and boxes/cans, have children match cans/boxes to tracing, hunt for circles and rectangles.*
2. **"Button Shapes"-** *Using a variety of buttons of all shapes, glue, paper, crayons/markers. Have children choose buttons and glue them to the paper to make a picture, then use crayons/markers to draw around it, ask child to describe picture, write down what child says, use ideas above for how to talk about shapes.*
3. **"Circle Time"-** *Using a large circle shaped item and other circle shaped items from home or classroom. Hold up large circle shaped item, trace the outline with your finger saying "This is a circle…this gores round and round." Ask children to close eyes and ask about circles they know, pass out other items and explore circles (describe, trace, stack, roll), make circles with arms, eyes, mouth, summarize attributes of a circle.*
4. **"I Spy Shapes"-** *Using various types of shapes, place the shapes around the room. Name a shape and have children guess where the shape can be found, child who guesses correctly names the next shape, repeat.*
5. **"Match and Name Shapes"-** *Using 2 sets of shapes (each set a different color), starting with a few simple, familiar shapes, adding more as children progress. Hold up one simple shape and have child find match, name the shape, describe it, ask "how do you know it is a ____?", repeat with another shape.*

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